Exam. Code : 211001
Subject Code : 4954

M.Sc. Mathematics 1st Semester (Batch 2021-23)

ALGEBRA—I

Paper—MATH-553

Time Allowed—3 Hours] [Maximum Marks—100

Note :— Attempt **FIVE** questions in all, selecting at least **ONE** question from each section. The fifth question may be attempted from any section. All questions carry equal marks.

SECTION—A

1. (a) Let G be a group and a, b be elements of group G such that ab = ba. Let o(a) = m and o(b) = n. If gcd(m, n) = 1, then show that o(ab) = mn. 10

   (b) Prove that a subgroup of cyclic group is cyclic. 10

2. (a) Give an example of a group G having subgroups K and T such that K is a normal subgroup of T and T is a normal subgroup of G but K is not a normal subgroup of G. 10

(b) Let G/N be the Quotient group of G. Suppose that o(gN) is finite. Show that o(gN) divides o(g). Also show that g^m belongs to N if and only if o(gN) divides m. 10

SECTION—B

3. (a) Prove that G', the commutator subgroup of group G, is normal subgroup of G and if H is a normal subgroup of G such that G/H is an Abelian group then prove that G' is contained in H. 10

(b) Find all automorphisms of S_3. 10

4. (a) Prove that for n > 1, A_n is a normal subgroup of S_n and it is of index 2. 10

(b) State and Prove Second isomorphism Theorem. 10

SECTION—C

5. (a) State and Prove Sylow's first theorem. 10

(b) Prove that there is no simple group of order 36. 10

6. (a) Let G be a group of order p^n, p prime. Prove that G has non trivial center and if H is a proper subgroup of G of order p^{n-1} them H is a normal subgroup of G. 10

(b) Prove that a finite group G is solvable if and only if its composition factors are cyclic groups of prime orders. 10

SECTION—D

7. (a) Prove that there is 1-1 correspondence between the family of non-isomorphic abelian groups of order p^e, p prime, and the set P(e) of partitions of e. 10

(b) Let A be finite Abelian group of order n and let d be divisor of n. Prove that A contains a subgroup of order d. 10

8. (a) Prove that S_n is semidirect product of A_n and cyclic group generated by transposition (12). 10

(b) If G is a group of order pq, where p and q are distinct primes, and if G has a normal subgroup H of order p and a normal subgroup K of order q, then prove that G is cyclic. 10